FINITE ELEMENT ANALYSIS OF EXTERNAL BRAKE AS A CONTACT PROBLEM

by

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CERTIFICATE

This is to certify that the thesis entitled ," **FINITE ELEMENT ANALYSIS OF AN EXTERNAL BRAKE AS A CONTACT PROBLEM** " by HIMANSHU SHEKHAR '9110515' is a bonafide record of work done by him under my guidance and supervision and has not been submitted elsewhere for the award of a degree.

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HIMANSHU SHEKHAR

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LIST OF SYMBOLS

SYMBOLS		QUANTITY
Ω	:	Domain
Γ	:	Boundary
E	:	Young's modulus of elasticity
ν	:	Poisson's ratio
$\{\sigma\}$	:	Stress Vector
$\{\varepsilon\}$	:	Strain Vector
$\{N\}$	:	Shape function
J	:	Jacobi matrix
$[D]$	:	2-D Elastic constant matrix
$[K]$	:	Stiffness matrix
p	:	Contact pressure
$\{u\}$	:	Displacement matrix
$\{F\}$	:	Load matrix
$\{X\}$	:	Coordinate Vector
$[B]$	:	Derivative of shape functions
ξ, η	:	Local coordinates
r	:	Drum radius
α	:	Angle between x-axis and tangent at contact point

subscripts or superscripts		definitions
x	:	Value in x -direction
y	:	Value in y -direction
ζ	:	Value in local ζ direction
η	:	Value in local η direction
ne	:	Nodal values
A	:	Values for first body
B	:	Values for second body
e	:	Values for a typical element 'e'
s	:	Value in tangential direction
n	:	Value in normal direction

ABSTRACT

This work is a finite element analysis of an external braking system , to determine contact stresses and slip . The study is restricted to a static case and the body forces are neglected . To limit the study to two elastic bodies in contact , lever is eliminated from the analysis and suitable force boundary conditions are applied to the top surface of the shoe . The standard two - body algorithm , which has a fast convergence rate is employed . The results are compared with the relations used in the conventional brake design procedure . The effect on contact stresses and slip are studied for change in parameters like the total contact angle , the coefficient of friction , the hinge location etc .

Chapter 1

Introduction

1.1 Brakes

Man has gained speed in all aspects of life . Be it a vehicle , machinery , or any other equipment , high velocity is a necessity . But they cannot run continuously as a perpetual motion machine . So a device to control their motion is needed . Brake is a device which by means of frictional resistance is used to either retard or stop a rotating member .

Brakes are classified on the basis of actuating mechanism as Hydraulic [Hydrodynamic Brakes and Fluid Agitator],Electric [Generator and Eddy Current Brakes] and Mechanical [Shoe , Band and Block - Band] . Mechanical brakes are further classified into two groups according to the direction of braking force :-

(1) Radial Brakes [External and Internal]

(2) Axial Brakes [DIsc and Cone]

In this thesis, only mechanical braking system with external shoe is considered . A typical system is shown in fig 1.1 . The essential part is a lever , with fulcrum at "F" to which shoe "S" is bonded . By applying external force "W" at the end of the lever , the shoe is brought in contact with the drum "D" an

braking action takes place . All necessary dimensions are shown in fig. 1.1 . When the drum is rotating in clockwise direction the free body diagram of the lever - shoe assembly is shown in fig. 1.2 . "N" is the resultant normal force and "μN" is the frictional resistance due to braking .

As the moments of frictional force and the external load "W" have the same direction , the frictional force is assisting the external load in the operation of braking . this type of brake is called self - energizing . If the direction of rotation in fig. 1.1 is reversed , the frictional force force will oppose the externally applied load as far as the moment about fulcrum is concerned . This is a de - energizing system of braking .

When the moment of all the forces about fulcrum is equated to zero , we get

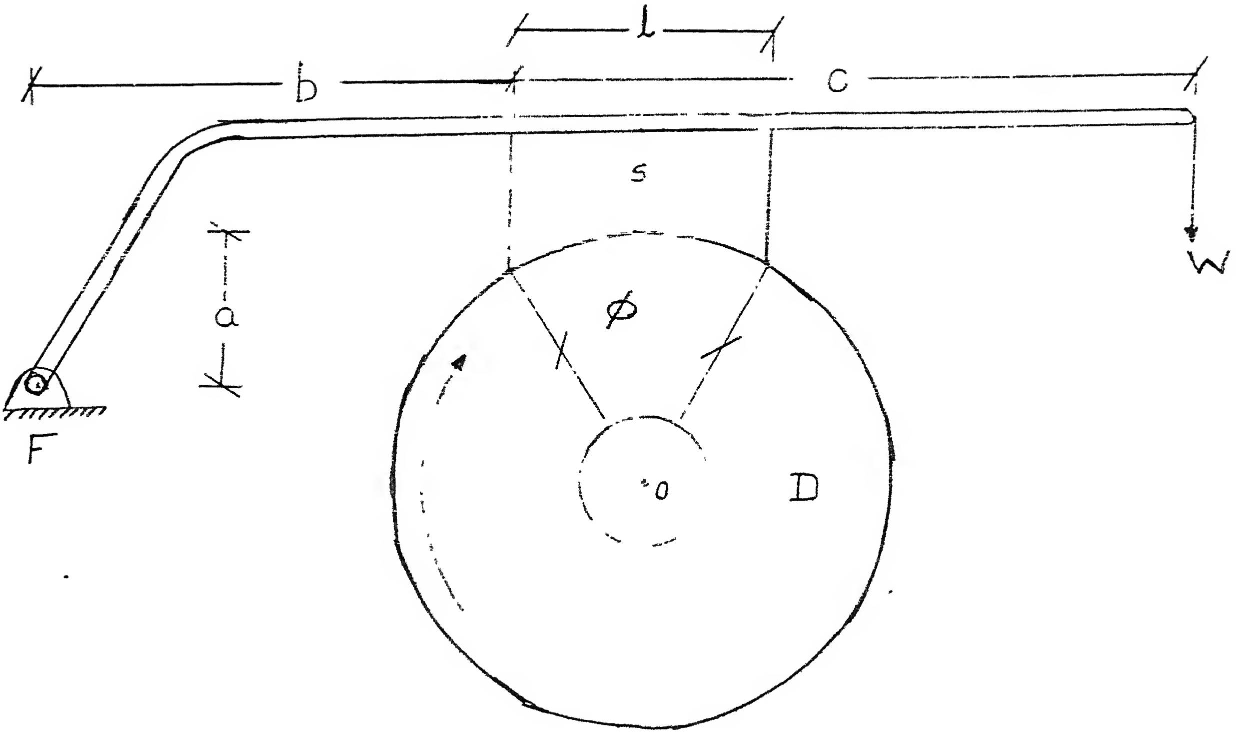
$$W = \frac{N(b+l/2-\mu a)}{b+c} \tag{1.1}$$

For b >> l/2

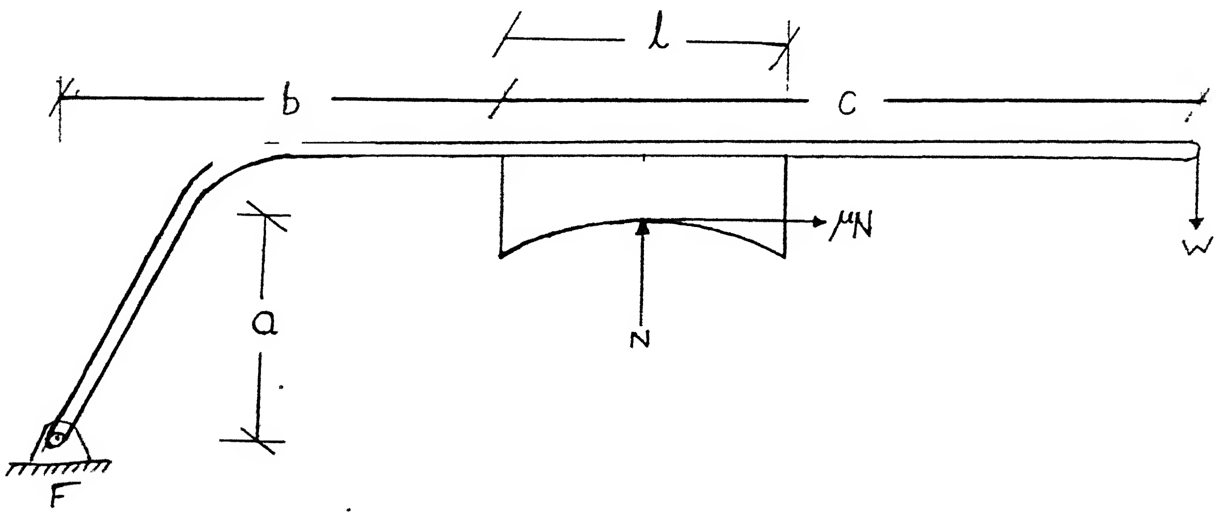
$$W = \frac{N(b-\mu a)}{b+c} \tag{1.2}$$

When $(b-\mu a) < 0$, "W" becomes negative . This situation is called self - locking . In self - locking brakes , external load is not applied to brake the drum . The shoe is merely allowed to come in contact with the drum . The frictional force generated by the contact itself is sufficient to brake the system .

A brake should be self - energizing to reduce the external



1.1 Long-Shoe External Braking System



1.2 Free Body Diagram Of Lever

load , But it should not be self - locking to avoid stoppage of the drum if , by accident , the lever falls on the drum .

The factors , which affect the performance of a braking system are as follows :-

(1) Coefficient of friction between the braking surfaces.

(2) Ability of shoe material to dissipate heat .

(3) Area of contact surface .

(4) Average and maximum pressure on the braking or contact surface

(5) Rate of wear .

(6) Velocity of the drum periphery .

To control the above factors for the favorable outcome , the major governing criteria are the selection of the shoe material and the braking or contact area of the shoe . The material with a high dynamic coefficient of friction is normally selected so that it can generate the resisting torque necessary for the braking action .

When a brake operates , heat is generated due to the presence of friction and slip at the drum - shoe interface . The generated heat depends on the coefficient of friction , the pressure distribution on the braking surface , the velocity of drum periphery , the inter facial slip and the area of braking surface . Since the temperature rise affects adversely the strength , the coefficient of friction and other characters of the shoe material , it should have high heat dissipating capacity

Sometimes it becomes necessary to provide some cooling arrangement.

The braking or the contact area of the shoe is selected so that the maximum pressure does not exceed the safe value for the shoe material . The maximum pressure also depends on the rate of wear . Velocity of drum periphery is , in general , not under the control brake designer but is a given parameter .

1.2 Brake Design Procedure

The primary purpose of a brake is to absorb energy and then dissipate it without permitting any appreciable temperature rise that can damage either the shoe or the drum surface . The brake design procedure , similar to designing any other mechanical device , starts with the establishment of general requirement for which the brake is to be designed . The primary requirement is the generation of braking torque sufficient to balance the drum torque for the given velocity .

The traditional method of brake design uses the following equation to relate the braking torque "T" with design variables like the coefficient of friction "μ" , the maximum pressure "p_{max}" , the radius of the drum "r" , the shoe width "b" and the arc of contact (Juvinall [1]) :

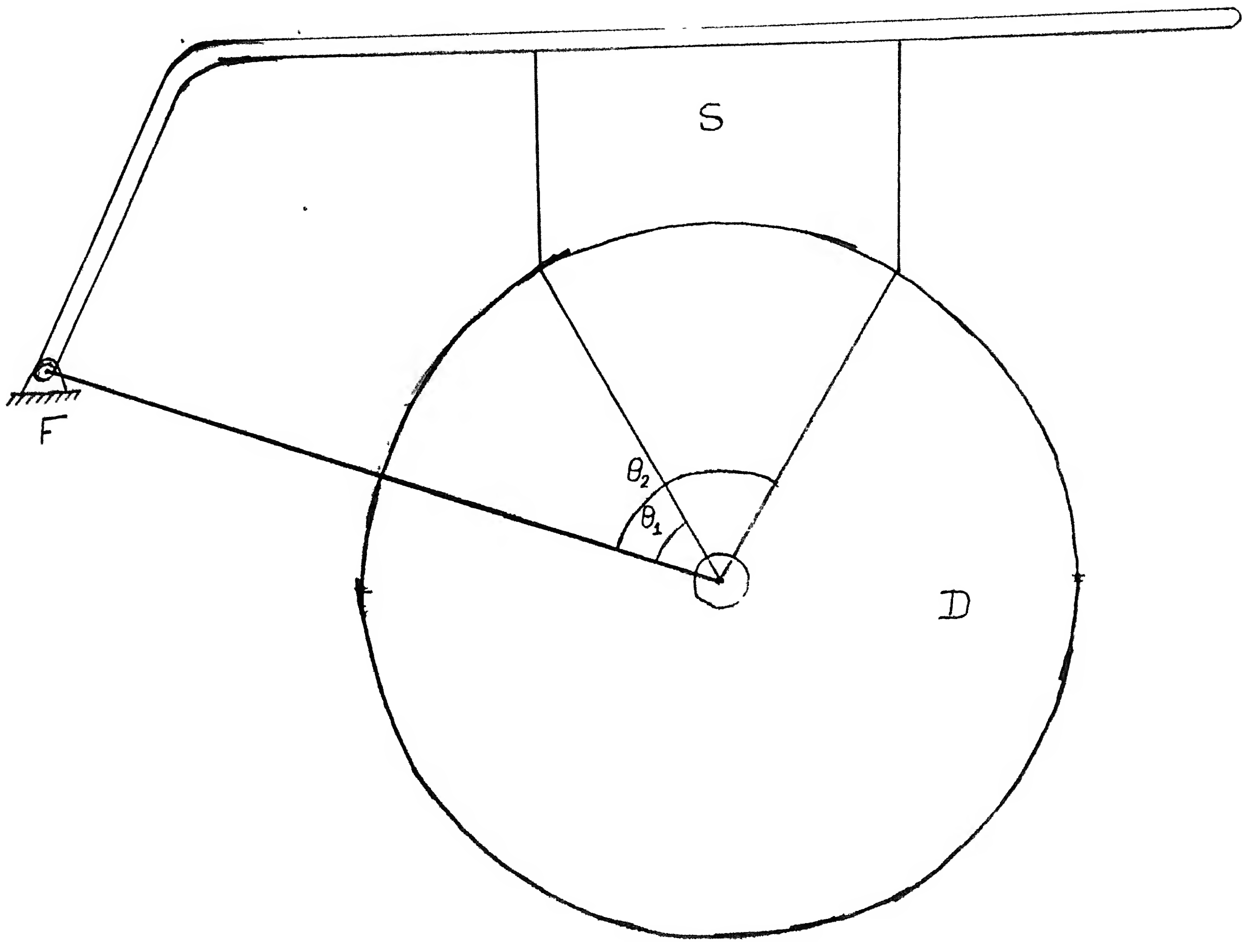
$$T = \frac{r^2 \mu\, b\, p_{max}\, (\cos\theta_1 - \cos\theta_2)}{(\sin\theta)_{max}} \qquad (1.3)$$

Here θ is the angle that a radial line on the drum makes with the line joining the drum center to the fulcrum and θ_1 and θ_2 are it's values at the contact points (fig. 1.3) . This formula is based on many idealizations . It is assumed that the pressure on the braking surface is proportional to the normal wear . Further , while calculating the normal wear , it is assumed that everything except the brake lining has infinite rigidity . This gives the following distribution of pressure along the contact surface .

$$p = p_{max} \frac{\sin\theta}{(\sin\theta)_{max}} \qquad (1.4)$$

First a suitable material for the shoe (and the drum if the selection of the drum material is in the jurisdiction of the brake designer) is chosen . Once the material is selected , the coefficient of friction "μ" between the mating surfaces and the capacity of the shoe material in terms of maximum allowable pressure "p_{max} " are known from the data books . Then a suitable fulcrum for the brake lever is chosen .Finally for the given drum of radius "r" and width "b" , the arc of contact (θ_1 - θ_2) is calculated iteratively from eqn. 1.1 .

The value of μ and p given in the data book are usually



1.3 Drum -- Shoe Assembly

for normal operating conditions , when it is assumed that the heat generated due to friction will be dissipated . Since both the coefficient of friction as well as the capacity to withstand pressure decrease with temperature , it is important to estimate the temperature at the friction - surface during braking and calculate the value of "μ" and "p " corresponding to the actual temperature . Unfortunately , it is quite difficult to calculate the temperature .

When the designer doesn't have temperature data available , he must rely on the principle that the rate of wear is proportional to the rate at which energy is absorbed by the friction surfaces (Phelan [2]) . This is because the temperature is also proportional to the rate of energy absorbed , which using the principle , the capacity of the lining material is expressed in terms of the product μpV , the rate of energy absorption per unit area . Dubey and Suresh [3] have suggested an empirical approach for two - wheeler's brake design in which the contact area and the drum diameter are related to the torque based on the study of the two - wheelers using new technology .

It is quite clear that the assumptions on which the eqn. 1.3 is based are too simplistic . For an optimum brake design , the rate of wear and the temperature distribution at the friction surface should be evaluated more accurately . Both these quantities depend on the rate of friction work which in turn

depends on the pressure distribution and the slip at the friction interface . But the pressure distribution and slip depends on the coefficient of friction and the contact geometry which are influenced by temperature and wear . Because of this the inter dependence . the procedure has to be iterative and incremental . It is obvious that for a brand new brake for which the wear is zero , the pressure and slip at the friction surface will be governed solely by the material properties of the shoe and drum , the coefficient of friction and the initial contact geometry . Once the pressure and the slip are known at the friction surface , the temperature at the initial instance can be determined using appropriate conduction equation . It can be used to modify the material properties and the coefficient of friction for the next iteration . The iteration can be continued till we get the convergence values of the pressure , slip and temperature . Next , the rate of wear at the initial instance can be calculated using appropriate diffusion equation . Using this information , we can modify the contact geometry for the calculation of the next instance .

In this thesis , the problem of determination of pressure and slip only for the brand new brake is undertaken . It is treated as a contact problem between two elastic bodies with friction . The external loading is both normal and tangential and the initial contact surface is circular .

1.3 Literature Survey

Several investigators have attempted to determine stresses and displacements in two elastic bodies having a curved contact surface . In three dimensions , the problem of contact of two elastic bodies was first formulated and solved by **Hertz** [4] under several restrictive assumptions . Research work based on Hertz's theory up to 1982 has been reviewed by *Johnson* [5] . Hertz assumed that the geometry of general curved surfaces in the vicinity of contact can be described by quadratic terms only . Then the geometric compatibility condition leads to an elliptical contact area . Neglecting friction and assuming that the bodies in contact deform as though they were elastic half - spaces , Hertz used the Boussinesq solution to deduce that the contact pressure distribution has to be elliptic to produce an elliptic contact area . Hertz evaluated the dimensions of the contact area and the stresses at the contact surface but presented only a speculative sketch of the sub surface stresses . The stresses beneath a general elliptical contact were analyzed first by **N. M. belajet** in two papers in 1917 and 1929 (see ref.5) . It is observed that maximum shear stress occurs at a depth of (0.57a) for a circular contact of radius 'a' .

From the Hertz's theory of contact stresses , where the

contact is over a finite area , the case of two - dimensional contact between two cylinders can be obtained as a limiting case by allowing the contact area to become infinitely long in one direction . However , it is easier to do it directly as shown by **Poritsky** [6] . He also evaluated the internal stresses due to tangential traction at the surface . He observed that , for a static contact , slipping occurs outside the central zone of length (2b') which is given by

$$b' = b \sqrt{1 - Q / (\mu P)} \qquad (1.5)$$

Here b is the semi - contact length , μ is the coefficient of friction and Q and P are respectively the tangential and normal loads such that $Q < (\mu P)$. On the other hand , for a rolling contact , the sticking region is no longer centrally located but is adjacent to the leading edge of the contact .

For a three - dimensional sliding contact $(Q = \mu P)$, the internal stresses due to combined effect of normal pressure and tangential traction were obtained by **Bryant & Keer and Sackfield & Hills** (see ref.5) . For a static contact , **Cottaneo** (see ref.5) showed that , to avoid slip the tangential stress at the boundary of contact area has to be infinite . Thus , some slip is inevitable at the edge of contact area even under the action of the smallest tangential load . Like a two - dimensional problem , here also the no - slip zone is a central region . For an elliptical contact area of semi - axis a and b , the semi - axes

a' and b' of the no - slip ellipse are given by

$$a' = a[1 - Q/(\mu P)]^{1/3} \quad , \qquad b' = b[1 - Q/(\mu P)]^{1/3} \qquad (1.6)$$

where μ , Q and P are as defined by eqn. 1.5 . The annular zone of 'micro - slip' penetrates further into the contact area with increasing Q until , when $Q=\mu P$ the whole contact slides . Being a dissipative process , the micro - slip has the effect of making the state of contact stress dependent upon the history of loading.

The effect of friction is to bring the point of maximum shear stress closer to the surface . For values of μ ,0.3, the maximum shear stress occurs at the surface .

Another important analytical method to solve contact problems (with or without friction) is to formulate it in terms of integral equations . **Muskhelishvili** [7] and **Gladwell** [8] solved some contact problems using this approach . Other notable solutions which use the method of integral equations are those of **Weitsman** [9] for unbounded contact between a plate on the elastic half - space , **Prasad & Dasgupta** [10] for rectangular block compressed by rigid planes and **Spence** [11] for indentation of elastic half - space by an axisymmetric punch .

When contact geometry cannot be described adequately by the quadratic terms , it becomes necessary to use numerical methods . Two approaches have been proposed . In the first approach , the conditions of displacement compatibility are supplied to an

assumed contact area and the equilibrium equations are solved to find the contact stresses . If near the edges , the normal stresses come out to be tensile , that region is excluded from the contact area in subsequent iterations . Further , if the shear stresses exceed the product of coefficient of friction (μ) and the normal stresses (t_n) , the tangential compatibility condition is replaced by the slip condition in subsequent iterations . The iterations are continued till the normal stresses become compressive and the shear stresses become less than or equal to μt_n . The second approach is based on the principle that the pressure distribution and the area of contact will be such as to minimize the total elastic strain energy , subject to no interference outside the contact and positive pressure inside . When friction is present at the contact , one needs an additional minimization principle to find the slip . The advantage of the method is that the process of minimization can be carried using mathematical programming .

Many investigators have adopted the technique of finite element method to solve the elastic contact problems . Here only a few typical papers are reviewed . The papers which use the first approach are discussed first . **Ohte** [12] has proposed a method in which the stiffness matrix is modified to eliminate the reaction forces at the contact nodes based on stick - slip contact conditions . On the other hand , **Sachdeva and Ramakrishnan** [13]

have used the flexibility matrix to satisfy the frictional contact conditions . The flexibility matrix is obtained by inverting the stiffness matrix , which is first condensed to eliminate all the nodes except those which are likely to be in contact and those where external forces act . **Gautam et al** [14] have compared the method of Ohte and Sachdeva & Ramakrishnan and concluded that in problems involving no symmetry of geometry or loading , the second method is more economical in terms of memory and solution time requirements . Whereas proportional loading has been assumed in both these methods , **Gaertner** [15] has used incremental loading . Triangular elements with 6 dof (normal displacement , tangential displacement , rigid rotation , tangential strain , normal stress and tangential stress) have been used . Use of stress components as dof facilitates adoption of any frictional law and not just Coulomb's law . Elasto-plastic frictional contact problem has been treated by **Valliappan et al** [16] with incremental loading . **Schreppers et al** [17] have presented a finite element formulation for a large sliding contact . For such a problem , criterion to identify sets of actual contacting material points constitutes an important aspect of the contact algorithm . **Dandekar and Conant** [18] have used the boundary element method to solve frictional contact problem.

In the above - mentioned works , the contact conditions have been applied by modifying either the stiffness matrix or the

flexibility matrix . However , in the last dozen years or so , in most of the papers , the contact conditions have been treated as additional constraints and lagrange multiplier or penalty function method have been used to satisfy them . **Okamoto and akazawa** [19] have used the method of lagrange multiplier to solve frictional contact problem with incremental loading . **Chaudhary and Bathe** [20] have developed a general algorithm for three - dimensional dynamic contact problem using lagrange multipliers . **Stadter and weise** [21] have developed a penalty method with the use of 'gap' elements where the gap element stiffness is taken as the penalty number . The stiffness is varied to account for regions of contact and separation . A gap element with friction was used by **Mazurkiewicz and Ostachowicz** [22] . **Mottershead et al** [23] have reviewed the literature on the use of lagrange multiplier or penalty function methods to frictional contact problem with incremental loading and presented a general finite element approach for the treatment of contact problems . The mathematical foundation of lagrange multiplier and penalty function methods by finite element technique is studied in **Kikuchi and Oden** [24] . Unlike the most problems in elastic stress analysis , the contact problems are posed as variational inequalities owing to the unilateral form of the displacement constraints .

The method of mathematical programming to non - Hertzian friction less contact problem was applied first by **Kalker and**

Ranen [25] . Hung and Sauxe [26] have used finite element method along with mathematical programming technique to solve friction less contact problem .Recently , Mahmoud et al [27] have used incremental convex programming (along with finite element technique to solve friction less contact problem . There does not seem to be any solution of frictional contact problem using the technique of mathematical programming .

All the above papers have placed emphasis on presenting a general methodology with a few illustrative problems rather than solving engineering problems of practical interest . Needless to say that there are no studies of the contact stresses in brakes either analytical or numerical.

1.4 Objective And Scope of Present Work

In a brake shoe -drum assembly , one contact surface is convex (i.e. that of drum) while the other is concave (i.e. that of shoe) . Therefore one cannot use Hertz's solution . In fact , it would give infinite contact length and zero contact pressure as the radii of curvature are equal and opposite . As stated in the section 1.3 , there is no analytical solution . Therefore , a finite element method is used for determination of the contact pressure and slip at the shoe-drum interface . The problem is idealized as two - dimensional (plain stress) . The distribution of the external loading on the shoe is calculated from the

assumption that it is proportional to the deflection of the brake lever as it rotates to bring the shoe in contact with the drum . The deflection is calculated from the geometrical compatibility .

The iterative algorithm proposed by sachdeva and Ramakrishnan [13] has been used expect for one difference . Whereas the contact condition have been applied to flexibility matrix in reference [13] , here they have been applied to stiffness matrix only . Like in reference [13] , the size of the global stiffness matrix has been reduced by skyline storage and condensation . The iteration algorithm converges quite fast and gives nodal displacements on convergence . The difference in the tangential displacements of the shoe and the drum at the contact surface gives the slip . To find the contact stresses ,first the stress tensors evaluated at the gauss points are extrapolated to find their values at the boundary points . Then the stress vector for each body is evaluated and the magnitude of the normal and tangential components of the two stress vectors are averaged .

The contact pressure distribution and the maximum value are compared with the corresponding equations of the traditional method of brake design . The parametric study is carried out to study the effects of contact angle , the coefficient of friction and the location of hinge on the contact pressure and the slip .

The present analysis is applicable to new brakes more effectively when wear is less . It can be extended further to find

the wear analysis . Distribution of external forces on the shoe boundary has been derived from the geometry of the drum - shoe assembly . The process nowhere uses any incremental loading procedure ,where final result depends on the accuracy with which the intermediate results converge . Instead full external load is acting on the shoe from the first iteration .

1.5 Plan of Thesis

Finite element formulation and sub structuring are discussed in chapter 2 . It also gives procedure for calculation of stress . Chapter 3 deals with discritisation ,boundary condition and results with discussions . The conclusion and scope for further work are presented in chapter 4.

CHAPTER 2

Mathematical Formulation Of Contact Problem

In this chapter , finite element algorithm for elastic bodies in contact is presented . The main feature of the algorithm are sub structuring of the conventional stiffness matrix to reduce it's size and the modification of the condensed stiffness matrix to take care of the contact conditions. Finally , the procedure for calculation of the contact stress is discussed as a post - processing phenomena .

2.1 Finite Element Formulation

A body can be specified by the domain Ω surrounded by the boundary Γ . The whole boundary of the body is divided into two parts :-

(a) Boundary where displacements are prescribed (Γu)

(b)Boundary where surface traction T are prescribed (Γf)

Fig. 2.1 shows a plate - like body of thickness h . For such a body plain stress situation prevails and stress - strain relation in vector form is given by

$$\{ \sigma \} = [D] \{ \varepsilon \} \tag{2.1}$$

where $\{ \sigma \} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$

$\{ \varepsilon \} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$

$$[D] = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

for plane stress

E = Young's modulus of elasticity

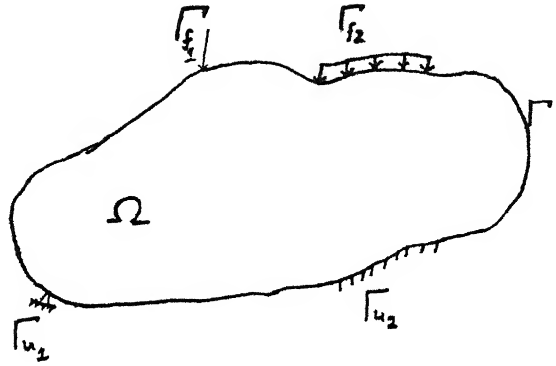
ν = Poison's ratio

Let the whole body is divided into four nod ed quadrilateral elements , in which node numbering scheme is as shown in figure 2.2 . For a any element ,displacement at any point can be expressed in terms of displacements at the nodes of the element . i.e.

$$\{ u \}^e = [N] \{ u \}^{ne} \tag{2.2}$$

where $\{ u \}^e$ = displacement vector at any point of a element

$= \{ u_x^e \; u_y^e \}^T$

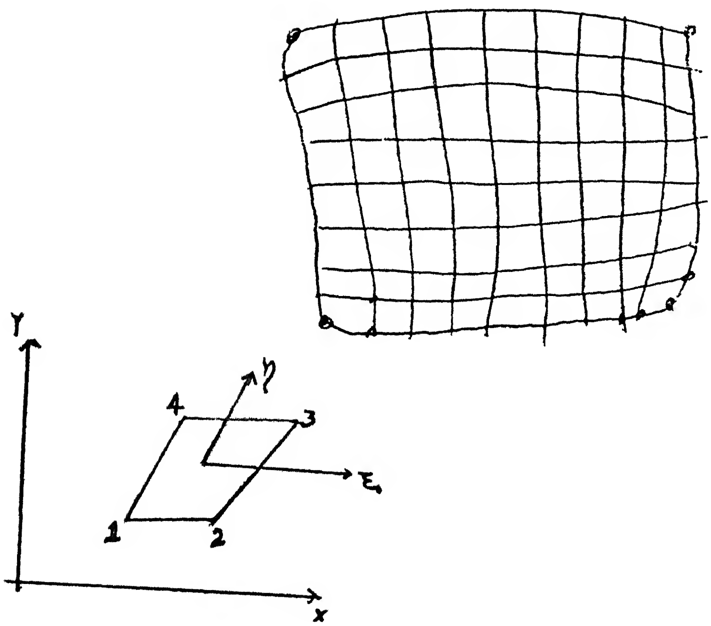


$$\Gamma_f = \Gamma_{f_1} + \Gamma_{f_2} + \ldots.$$

$$\Gamma_u = \Gamma_{u_1} + \Gamma_{u_2} + \ldots.$$

$$\Gamma = \Gamma_f + \Gamma_u$$

2.1 Plate Like Body



2.2 Discritisation And Node Numbering

$\{ u \}^{ne}$ = vector containing the nodal values of the displacement components

$= \{ u_{x1}, u_{y1}, u_{x2}, u_{y2}, u_{x3}, u_{y3}, u_{x4}, u_{y4} \}^T$

and [N] = matrix containing shape functions

$$= \begin{bmatrix} N_1 & 0 & \vdots & N_2 & 0 & \vdots & & \vdots & N_4 & 0 \\ 0 & N_1 & \vdots & 0 & N_2 & \vdots & & \vdots & 0 & N_4 \end{bmatrix}$$

Fig. 2.2 also shows the node numbering scheme for a particular element . For each element , local coordinate system (ξ,η) is chosen such that the local coordinates of the nodes 1, 2, 3, and 4 are respectively (-1,-1) , (1,-1) ,(1,1) and (-1,1) . Then shape functions are defined as

$$\left.\begin{aligned} N_1 &= \frac{1}{4}(1-\xi)(1-\eta) \\ N_2 &= \frac{1}{4}(1+\xi)(1-\eta) \\ N_3 &= \frac{1}{4}(1+\xi)(1+\eta) \\ N_4 &= \frac{1}{4}(1-\xi)(1+\eta) \end{aligned}\right\} \qquad (2.3)$$

The global coordinates x and y of a point in an element are related to the local coordinates ξ and η by the equation

$$\{ X \}^e = [N] \{ X \}^{ne} \qquad (2.4)$$

Where $\{ X \}^e$ =coordinates of a point in an element

$= \{ x^e \; y^e \}^T$

$\{ X \}^{ne}$ =coordinates of the nodal point of the element

$= \{ x_1 \; y_1 \; x_2 \; y_2 \; x_3 \; y_3 \; x_4 \; y_4 \}^T$

[N] = Shape function as defined in equation 2.2

In finite element formulation , relationship between strain displacement is expressed as :-->

$$\varepsilon_x = \frac{\delta ux}{\delta x} \; ; \quad \varepsilon_y = \frac{\delta uy}{\delta y} \; ; \quad \gamma_{xy} = \frac{\delta uy}{\delta x} + \frac{\delta ux}{\delta y}$$

$$\text{so } \{ \varepsilon \} = \{ \varepsilon_x \; \varepsilon_y \; \gamma_{xy} \}^T = \begin{bmatrix} \frac{\delta}{\delta x} & 0 \\ 0 & \frac{\delta}{\delta x} \\ \frac{\delta}{\delta y} & \frac{\delta}{\delta x} \end{bmatrix} \begin{Bmatrix} ux \\ vy \end{Bmatrix}^e \qquad (2.5)$$

By using eqn. 2.2 it becomes

$$\{ \varepsilon \} = [B] \{ U \}^{ne} \text{------} \qquad (2.6)$$

$$\text{where} \quad [B] = \begin{bmatrix} \frac{\delta}{\delta x} & 0 \\ 0 & \frac{\delta}{\delta x} \\ \frac{\delta}{\delta y} & \frac{\delta}{\delta x} \end{bmatrix} [N]$$

The potential energy of the system is the difference of the strain energy U and the work W of external force . Thus

$$\Pi = U - W \qquad (2.7)$$

$$\text{where} \quad U = \frac{1}{2} \cdot \int_\Omega \{ \varepsilon \}^T \{ \sigma \} \; h \; dA$$

$$\text{and} \quad W = \int_{\Gamma f} \{ u \}^T \{ T \} \; h \; dl$$

Here the body forces have been neglected . Consider a typical element 'e' whose area is Ω_e and which has the boundary Γf_e as a part of Γf . Note that for internal elements and for elements which do not have any boundary as Γf has no contribution in W .

Substituting eqns. (2.1) and (2.6) into (2.8), we get the total potential energy π_e for the element 'e' as

$$\pi_e = \frac{h}{2}\int_{\Omega_e} \{ u \}^{ne^T} [B]^T [D] [B] \{ u \}^{ne} \, dA - \int_{(\Gamma f)_e} \{ u \}^{ne^T} [N]^T \{ T \} \, dl \qquad (2.8)$$

For the element to be in equilibrium, the potential energy should be minimum. Setting the first derivative of π_e to zero, we get

$$[K]^e \{ u \}^{ne} = \{ F \}^e \qquad (2.9)$$

where

$[K]^e$ = stiffness matrix for one element

$$= \int_{\Omega_e} [B]^T [D] [B] \, dA$$

and $\{ F \}^e$ = load matrix for one element

$$= \int_{\Gamma f_e} [N]^T \{ T \} \, dl$$

In order to evaluate $[K]^e$ by numerical integration, it needs to be transformed to (ξ,η) coordinates by using the transformation eqn. (2.4). Then dA becomes equal to $| J | \, d\xi \, d\eta$, where $| J |$ is the determinant of the jacobian matrix given by

$$\text{JACOBI MATRIX} = \begin{bmatrix} \frac{\delta x}{\delta \xi} & \frac{\delta y}{\delta \eta} \\ \frac{\delta x}{\delta \xi} & \frac{\delta y}{\delta \eta} \end{bmatrix}$$

Here $[K]^e$ has been evaluated by 2 X 2 gauss - legendre numerical integration scheme . After calculating the elemental stiffness matrix , it's elements are transferred to the global stiffness matrix , which is stored in the skyline form . To accommodate distributed loading on the surface , $\{T\}$ on each side is expressed in terms of nodal traction values $\{T\}^{ne}$. Thus

$$\{T\}^e = [N]\{T\}^{ne} \tag{2.10}$$

where $[N]$ is given by eqn. (2.2) and

$$\{T\}^{ne} = \text{Nodal traction}$$
$$= \{T_{x1}, T_{y1}, T_{x2}, T_{y2}, T_{x3}, T_{y3}, T_{x4}, T_{y4}\}^T$$

Thus $\{F\}^e$ in eqn (2.9)becomes

$$\{F\}^e = \int_{\Gamma f_e} [N]^T [N] \{T\}^{ne} dl \tag{2.11}$$

The elemental load vector is also evaluated numerically by gauss - legendre integration scheme after the change of variable from (x,y) to (ξ,η) . After calculating the load vectors for all the elements , they are assembled into the global load vector .

2.2 SUB STRUCTURING

For contact problems , two bodies have to be analyzed simultaneously . This makes the size of stiffness matrix quite large , which requires more memory space on the computer as well as more computational time . To reduce the memory size as well as

computational time , condensation of global stiffness matrix is carried out . As a first step in the processes of condensation the nodes are divided into three categories

(1) Nodes where nonzero external load is applied or nodes which are in contact . They are called nodes of type 1 and are numbered first .

(2) Nodes for which the specified displacements are zero . They are called nodes of type 3 and are not numbered .

(3) All nodes except those lying in the first two categories are called nodes of type 2 and are numbered after 1st types of nodes . They include internal nodes as well as those external nodes where the specified external forces are zero .

Consider the bodies 'A' and 'B' shown in fig. 2.3 . The application of the formulation of section 2.1 leads to

$$\left.\begin{aligned} [K^A]\{u^A\} &= \{F^A\} \\ [K^B]\{u^B\} &= \{F^B\} \end{aligned}\right] \qquad (2.12)$$

Partitioning $[K^A]$ according to the type of nodes,discussed above we have

$$\begin{bmatrix} [K^A_{11}] & [K^A_{12}] & [K^A_{13}] \\ [K^A_{21}] & [K^A_{22}] & [K^A_{23}] \\ [K^A_{31}] & [K^A_{32}] & [K^A_{33}] \end{bmatrix} \begin{Bmatrix} \{u^A_1\} \\ \{u^A_2\} \\ \{u^A_3\} \end{Bmatrix} = \begin{Bmatrix} \{F^A_1\} \\ \{F^A_2\} \\ \{F^A_3\} \end{Bmatrix} \qquad (2.13)$$

As $\{u_s\} = \{0\}$, the first two set of equations can be written as

$$\begin{bmatrix} [K^A_{11}] [K^A_{12}] \\ [K^A_{21}] [K^A_{22}] \end{bmatrix} \begin{Bmatrix} \{u^A_1\} \\ \{u^A_2\} \end{Bmatrix} = \begin{Bmatrix} \{F^A_1\} \\ \{F^A_2\} \end{Bmatrix} \tag{2.14}$$

Similar equations are obtained for body 'B' also . Combining these equations ,we have

$$\begin{bmatrix} [K^A_{11}] & [0] & [K^A_{12}] & [0] \\ [0] & [K^B_{11}] & [0] & [K^B_{12}] \\ [K^A_{21}] & [0] & [K^A_{22}] & [0] \\ [0] & [K^B_{21}] & [0] & [K^B_{22}] \end{bmatrix} \begin{Bmatrix} \{u^A_1\} \\ \{u^B_1\} \\ \{u^A_2\} \\ \{u^B_2\} \end{Bmatrix} = \begin{Bmatrix} \{F^A_1\} \\ \{F^B_1\} \\ \{F^A_2\} \\ \{F^B_2\} \end{Bmatrix}$$

i.e.

$$\begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{21}] & [K_{22}] \end{bmatrix} \begin{Bmatrix} \{u_1\} \\ \{u_2\} \end{Bmatrix} = \begin{Bmatrix} \{F_1\} \\ \{0\} \end{Bmatrix} \tag{2.15}$$

Considering the second equation and eliminating the braces for ease in writing, following equation is obtained :-->

$$u_2 = - K^{-1}_{22} K_{21} u_1 \tag{2.16}$$

putting the value of u_2 in equation number 1 of (2.15), following equation is obtained

$$\hat{K} u_1 = F_1 \tag{2.17}$$

where

$$[\hat{K}] = [K_{11} - K_{12} K^{-1}_{22} K_{21}]$$

Thus the final matrix size is reduced to that of 1st types of nodes . The displacements of type 2 nodes can be calculated using eqn. (2.16) whenever needed . Note that while evaluating [$\hat{K}$] from eqn. (2.17) ,one has to extract [K_{11}] , [K_{22}] and [K_{12}] = [K_{21}]^T from the matrices [K^A] and [K^B] which are in skyline form .

2.3 Contact Conditions And Their Application

All the nodes except those in the possible contact region have either the known force or the known displacement ,But at the contact nodes , neither force nor displacement is known . So the sub structured stiffness matrix [$\hat{K}$] ,obtained in the previous section , cannot be inverted to get the displacements . To overcome this difficulty the sub structured stiffness matrix is modified with the help of contact conditions . As each contact node on the first body has a corresponding node on the second body and each node has two degrees of freedom , there are four equations for every node on the contact surface . So to make the stiffness matrix solvable , four contact conditions are needed .

The contact conditions are as follows :-

(1) The normal forces at the mating contact nodes are equal and opposite .

(2) The tangential forces are equal and opposite at the in -

contact nodes .

(3) If the nodes in contact are

(a) sticking, the tangential displacements are equal and in the same direction .

(b) slipping, the tangential forces at the node should be μ times the normal force , Where μ is coefficient of friction .

(4) The difference between the normal displacements is equal to the clearance between the contacting surfaces .

To apply the contact condition to the bodies shown in fig. 2.3 ,where node 'i' of body 'A' comes in contact with the node 'j' of body 'B' . (2i)th and (2i-1)th rows of [$\hat{K}^A$] and (2j)th and (2j-1)th rows of [$\hat{K}^B$] have to be modified . The details are described below .

(1) The mathematical statement of the first contact condition is

$F_{ni} = F_{nj}$,where ni and nj are the normal directions at the nodes 'i' and 'j' . In terms of (x,y) components the condition becomes

$$-F_{ix} \, Sin\alpha + F_{iy} \, Cos\alpha - F_{jx} \, Sin\alpha + F_{jy} \, Cos\alpha = 0$$

where 'α' is the angle made by the tangential direction s_i with the x-axis . To apply the condition (2i-1)th row of $\hat{K}$ is replaced by

-(2i-1)th row X Sinα + (2i)th row X Cosα

-(2j-1)th row X Sinα + (2j)th row X Cosα

and F(2j-1) is made zero .

(2) The second contact condition can be stated as $F_{is} - F_{js} = 0$ or in terms of the (x,y) components

$-F_{ix} \operatorname{Cos}\alpha - F_{iy} \operatorname{Sin}\alpha + F_{jx} \operatorname{Sin}\alpha + F_{jy} \operatorname{Sin}\alpha = 0$

To apply this condition (2i)th row of [$\hat{K}$] is modified by

(2i-1)th row times (-Cosα) + (2i)th row times (-Sinα) + (2j-1)th row times (Cosα) + (2j)th row times (Sinα)

and F(2j) is made zero .

It is to be kept in mind that after applying first contact condition , (2i-1)th row of [$\hat{K}$]gets modified . So (2i-1)th row of [$\hat{K}$] has to be stored for the purpose of applying the second contact condition .

(3) The third contact condition has two forms .

(a) If the node is sticking $u_{is} + u_{js} = 0$,or

$u_{ix} \operatorname{Cos}\alpha + u_{iy} \operatorname{Sin}\alpha - u_{jx} \operatorname{Cos}\alpha - u_{jy} \operatorname{Sin}\alpha = 0$. This condition is used to replace (2j-1)th row of [$\hat{K}$] . Thus all the elements of this row are made zero except the following

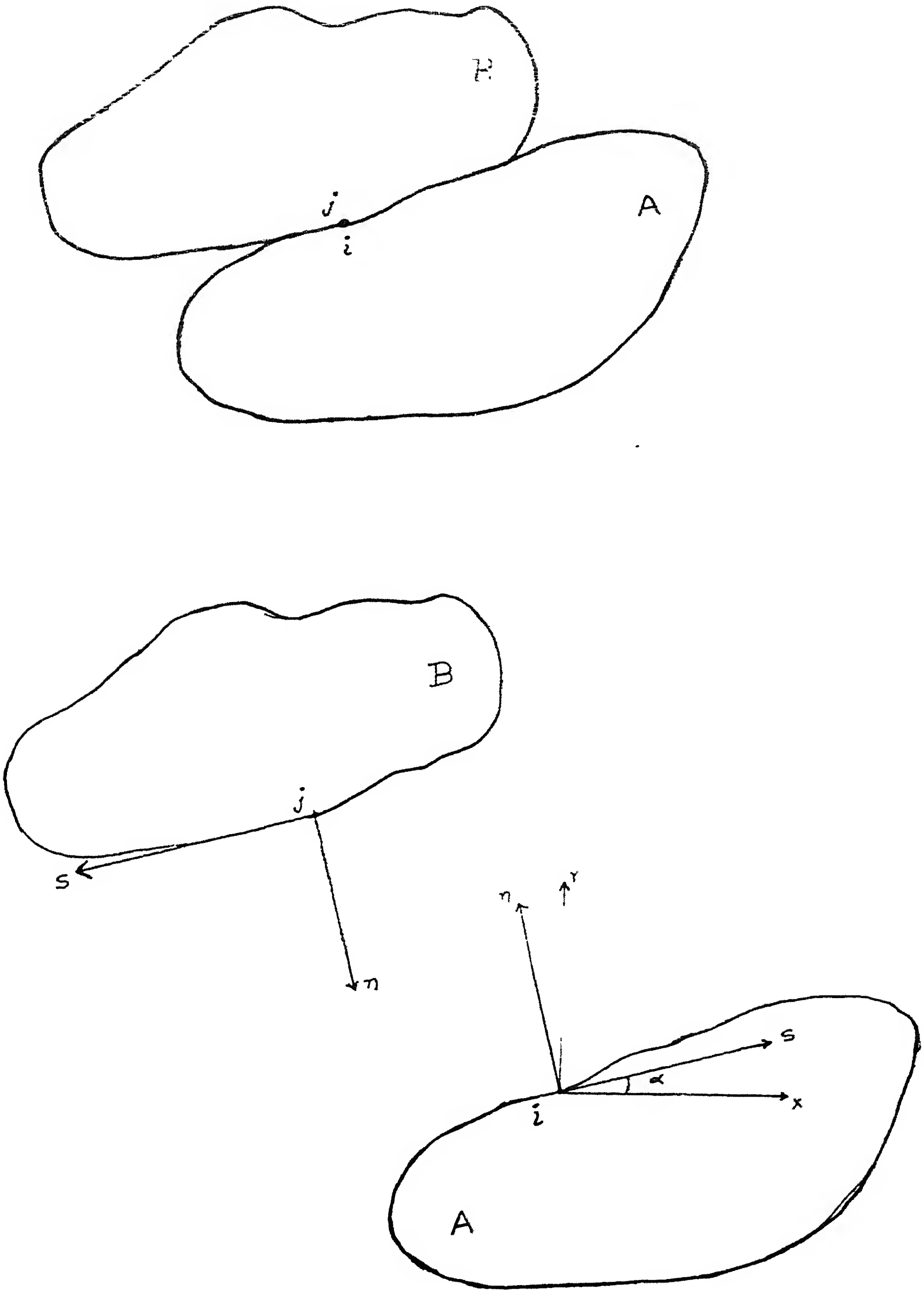
(2i-1)th element is made (Cosα) ,(2i)th element is made (Sinα) , (2j-1)th element is made (-Cosα) and (2j)th element is made (-Sinα) . Finally F(2j-1) is made zero .

(b) If the node is slipping , $|F_s| - \mu |F_n| = 0$. For the second body , this condition becomes

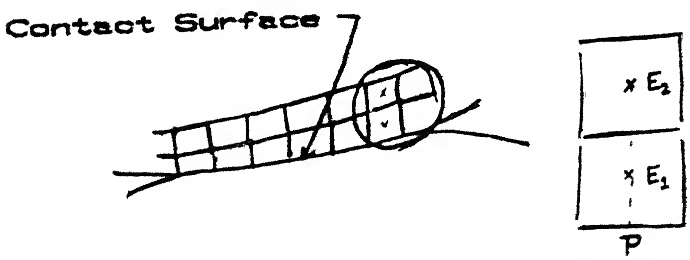
$F_{jx} (ss \times \operatorname{Cos}\alpha + sn \times \mu \times \operatorname{Sin}\alpha) + F_{jy} (ss \times \operatorname{Sin}\alpha - sn \times \mu \times \operatorname{Cos}\alpha) = 0$

.where '*ss*' and '*sn*' are respectively the signs of F_{js} and F_{jn} .

To apply this condition [$\hat{K}$] is replaced by



2.3 TWO ELASTIC BODIES IN CONTACT



2.4 Contact Stress Calculation

(ss X $Cos\alpha$ + sn X μ X $Sin\alpha$) times (2j-1)th row + (ss X $Sin\alpha$ - sn X μ X $Cos\alpha$) times (2j)th row . The right hand side is made zero .

(4) The fourth contact condition $u_{in} - u_{jn}$ = clearance or $-u_{ix} Sin\alpha + u_{iy} Cos\alpha - u_{jx} Sin\alpha + u_{jy} Cos\alpha$ = clearance is used to replace (2j)th row of [$\hat{K}$] . Thus all elements of the row are made zero except the following :-

(2i-1)th element is made ($-sin\alpha$), (2i)th element is replaced by ($Cos\alpha$) , (2j-1)th element changes to ($-Sin\alpha$) and (2j)th element becomes ($Cos\alpha$) . The right hand side is made equal to the clearance .

After modifying the stiffness matrix [$\hat{K}$] and the load matrix { F } as described above , the displacements of the nodes of type 1 can be obtained by solving the modified system . However , the number of contact nodes i.e. the contact length is not known a priori . Therefore the set of equations (2.17) have to be solved iteratively as described in the next section .

2.4 Solution Procedure

1 A certain number of nodes are assumed to be in contact . Further all of them are assumed to be sticking . The contact conditions are described in section 2.3 are applied to modify the matrix [$\hat{K}$] and { F } . The modified system is solved to find the

displacements of the nodes of type i . Then the nodal forces at these nodes are calculated .

2 The nodal forces are checked to determine the status of nodes for the next iteration .

If the normal force at a node becomes tensile (outward from the surface) ,it is deleted from the possible contact zone for the next iteration .

For the nodes remaining in contact the value of $|F_s|/|F_n|$ is compared with μ . If $|F_s|/|F_n|$ is greater than μ , it means the node slips and in the next iteration , the slipping contact condition [3(b) of section 2.3] is applied at these nodes .

3. The iterations are continued until all the normal forces in the contact zone became compressive and the ratio of tangential force to the normal force at all the contact nodes became equal to or less than the coefficient of friction .

Once the iteration are over , the stresses are computed as described in the next section .

2.5 Contact Stress Calculation

It is observed that when bilinear approximation is used for displacements , stresses are accurate at the center of the element . Therefore , first the stress components are determined at the center of elements in the layers adjacent to the contact surface .

The calculation of stress components is done using the eqn. (2.1) and eqn. (2.6) . Then the stress components at boundary points are obtained by extrapolation . For example , to find the stresses at point 'P' , extrapolation is done from the center of the element 'E1' and 'E2' (see fig. 2.4) . Then the surface tractions at the boundary points are determined from the equation

$$\begin{Bmatrix} T_x \\ T_y \end{Bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \begin{Bmatrix} \nu_x \\ \nu_y \end{Bmatrix} \qquad (2.18)$$

where ν_x and ν_y are the direction cosines of the outward normal vector to the boundary .Then the normal and tangential stress components (T_n and T_s) are obtained from

$$\left.\begin{aligned} T_n &= T_x \nu_x + T_y \nu_y \\ T_s &= T_x \nu_y - T_y \nu_x \end{aligned}\right\} \qquad (2.19)$$

The same procedure is adopted to find the normal and tangential stress component on the boundary of the second body . Finally magnitude of the normal and tangential stress components are averaged over the two bodies .

CHAPTER 3

RESULTS AND DISCUSSION

In this chapter , analysis of an external shoe brake system is carried out as a plain stress contact problem using the formulation of chapter 2 . The variations of normal and shear stresses as well as the relative slip are obtained along the contact surface . Comparison with the expression used in the conventional design procedure as well as parametric studies are also undertaken .

3.1 Modeling Of The Problem

The brake system is idealized as a contact problem between the shoe and the drum only . Since the lever has been completely eliminated from the analysis an important aspect of the modeling is the estimation of the distribution of the forces exerted by the lever on the shoe . To determine this distribution , we assume the following :-

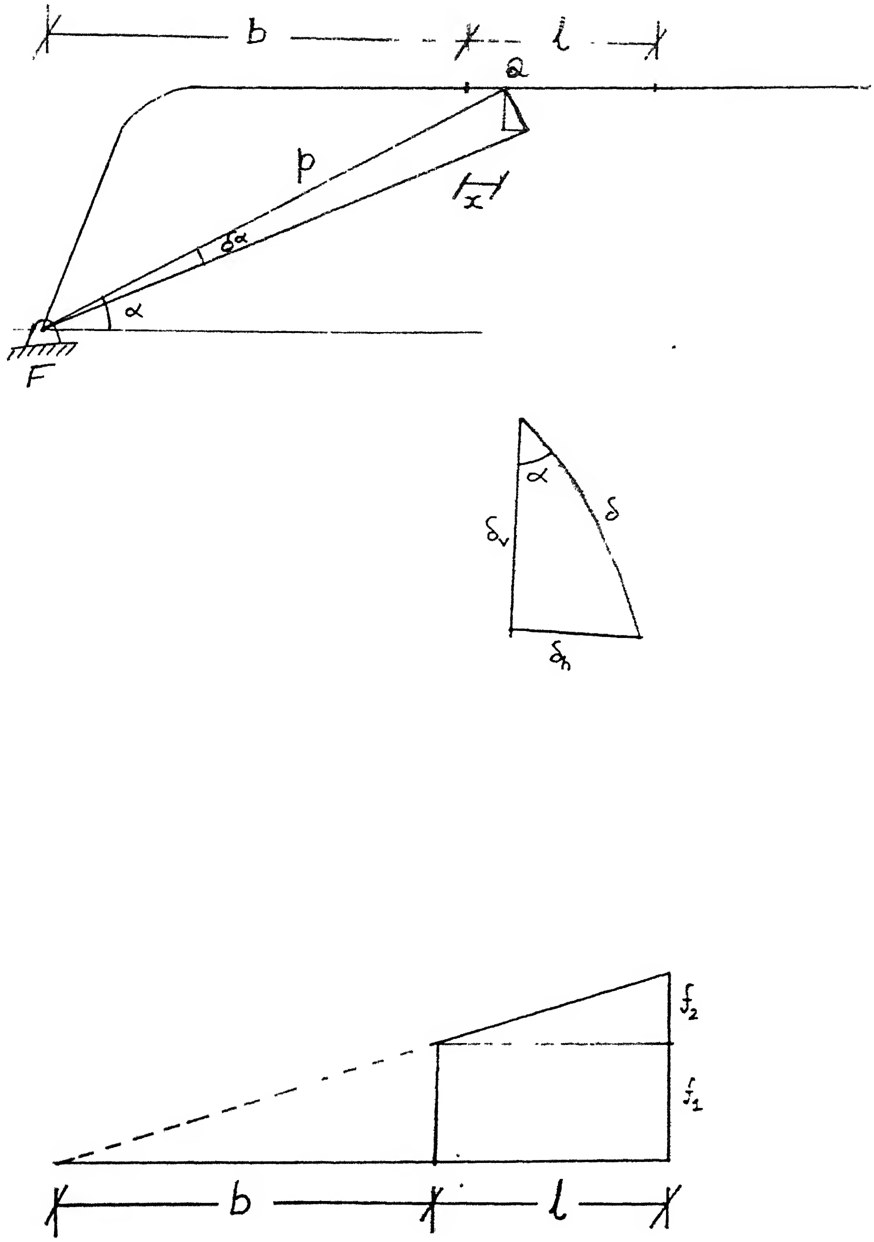
(i) The lever is rigid compared to the shoe .

(ii) As the lever rotates about the fulcrum to make the contact with the drum , the normal stress exerted by the lever on the drum is proportional to the vertical deflection of the shoe .

Consider the geometry of the lever shown in the fig. 3.1 . When the rigid lever rotates through $\delta\alpha$ about 'F' , the deflection of the point 'Q' on the lever is $p(\delta\alpha)$, where 'p' is the distance of the point from 'F' . If 'α' is the angular location of the point with the horizontal , then the vertical components δv of the deflection becomes $p(\delta\alpha)\text{Cos}\alpha$. Since $p\text{Cos}\alpha = b+x$, the horizontal distance of 'Q' from the fulcrum , the vertical deflection δv of the point becomes proportional to it's horizontal distance from the fulcrum , i.e.

$$\delta v \propto (b + x) \tag{4.1}$$

Since the normal stress exerted by the lever is assumed to be proportional to δv , it also becomes proportional to (b+x) . Thus , the variations of the normal stress becomes linear . The modeling of the actual variation by a linear function seems to be a better approximation than by a constant function (i.e. uniform distribution). To find the actual variation , one has to include the lever and consider it as a multiple - contact problem . This is likely to make the problem very complicated . It is felt that for a thick shoe , the derivation of the actual variation from the linear case may not affect the far away stresses at the shoe - drum contact , very much . Therefore , it is decided to use the linear variation for the normal stress . The distribution of the tangential force is assumed to be uniform . The better approximation for this distribution may be obtained by relating i



3.1 DISTRIBUTION OF THE NORMAL FORCE EXERTED BY THE LEVER ON THE SHOE

to the horizontal deflection of the shoe , but it is not done again felt that it may not affect the contact stresses very much

If f_1 and f_2 are the values of the normal stress at the tw end points as shown in fig. 3.1 , then for the case of linea variation

$$\frac{f_1}{f_2} = \frac{b}{b+1} \tag{4.2}$$

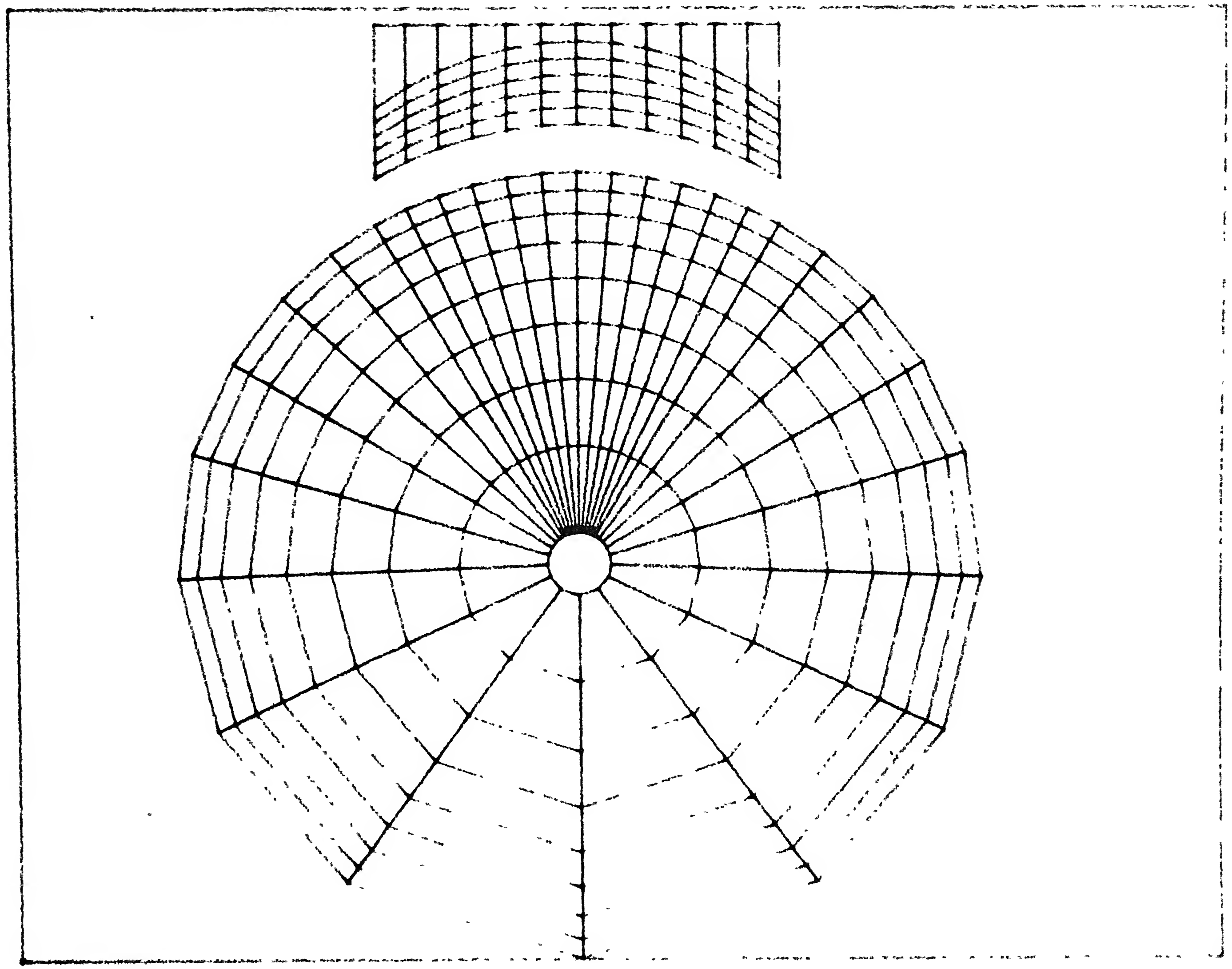
Further , if 'N' is the value of the normal force pr unit sho width required to generate the specified braking torque , then

$$N = 1/2\ (f_1+f_2)\, l \tag{4.3}$$

Solution of the eqns. (4.2) and (4.3) gives

$$\left.\begin{aligned} f_1 &= \frac{2\ N\ b}{(2\ b\ +\ 1)\ l} \\ f_2 &= f_1 + \frac{2\ N}{2\ b\ +\ 1} \end{aligned}\right\} \tag{4.4}$$

The distri tisa tion scheme for the fulcrum and the shoe is show in the fig. 3.2 . To solve the finite element equation of th assembly , all the rigid body modes have to be constrained . Fo the drum , it is done by assuming that the part , which is i contact with the shaft has zero displacements . Since , a fixe support gives rise to reactive forces , constraining any othe point will not be realistic . Further , this part of the drum i far away from the contact with the shoe . So it will not affec the contact stresses appreciably . The compatibility of the norma



3.2 DISCRETISATION OF DRUM AND SHOE

displacement between the drum and shoe removes some of the rigid body modes of the shoe . However , the tangential force on the shoe is equal to μ times the normal force . Therefore , the shoe is force to rotate about the drum axis . To constrain this rotation , the slip at the right - most contact point of the drum i.e. the actual slip at a point minus the slip of the right most point . To find the absolute slip , we need to add the slip of the right most point , which can be calculated from the velocity of the drum periphery.

3.2 Material Properties And Other Data

The following data , which is typical to a brake system is used in getting most of the results . In parametric studies the contact angle (ϕ) , the coefficient of friction (μ) and the hinge location (b) are changed .

Material properties

Drum —— cast iron .

Young modulus of elasticity $E_1 = 1.0 \times 10^5$ N/mm^2

Poisson's ratio ν_1 = 0. 25

Shoe —— sintered material

Young modulus of elasticity $E_2 = 6.0 \times 10^3$ N/mm^2

Poisson's ratio ν_2 = 0.3

For the sintered material on cast iron , the dynamic coefficient

of friction varies from 0.15 to 0.45 , whereas the capacity for maximum pressure varies from 1.030 N/mm^2 to 2.070 N/mm^2 . The following values are used .

Coefficient of friction μ = 0.3

Normal force on the brake N = 100.0 N/mm

Dimensions

Drum radius r = 125.0 mm

Shoe - thickness = 30.0 mm

Shaft diameter = 10.0 mm

Total contact angle of shoe and drum , ϕ = 60°

Lever dimensions [fig. 1.1]

a = 50.0 mm

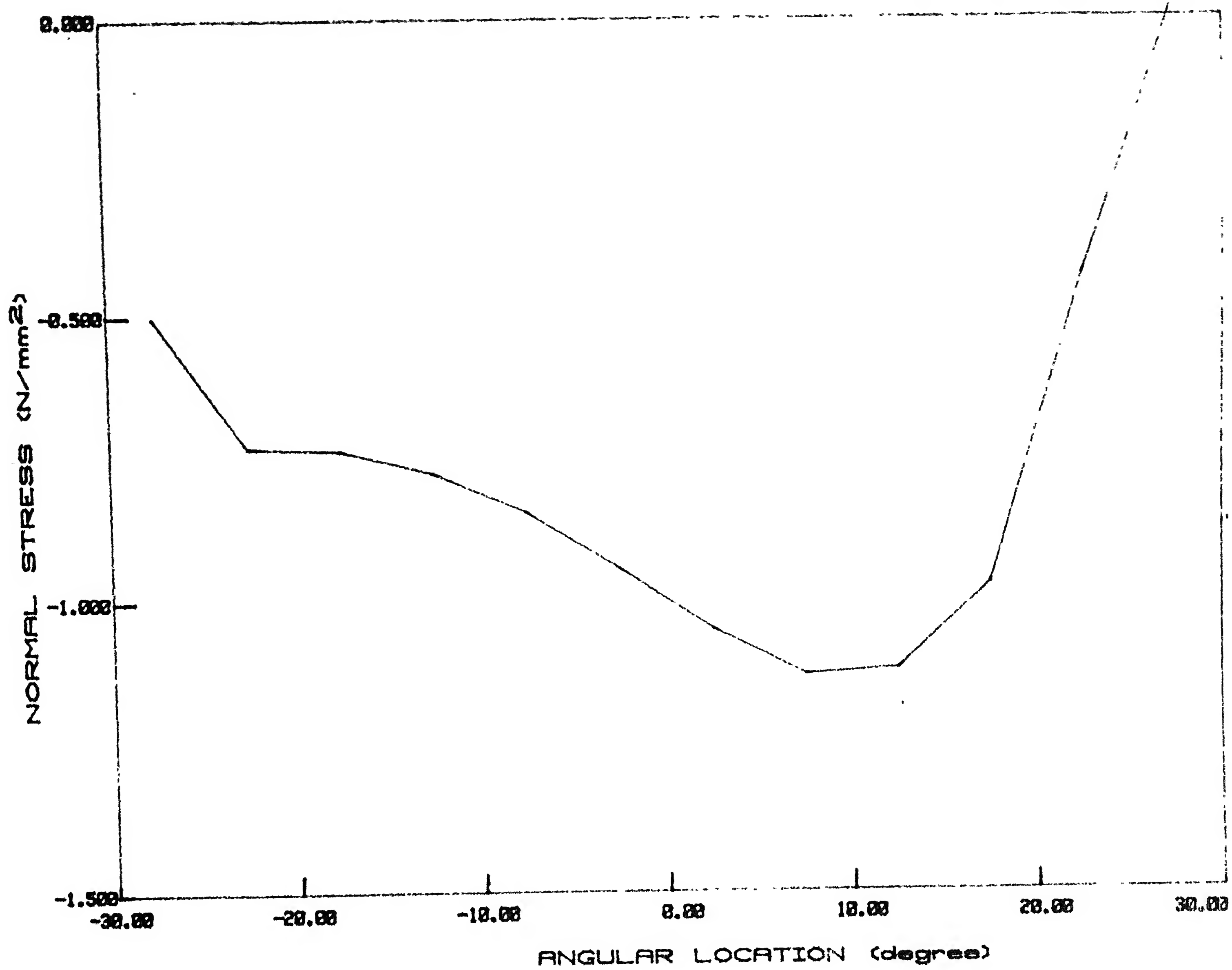
b = 200.0 mm

c = 250.0 mm

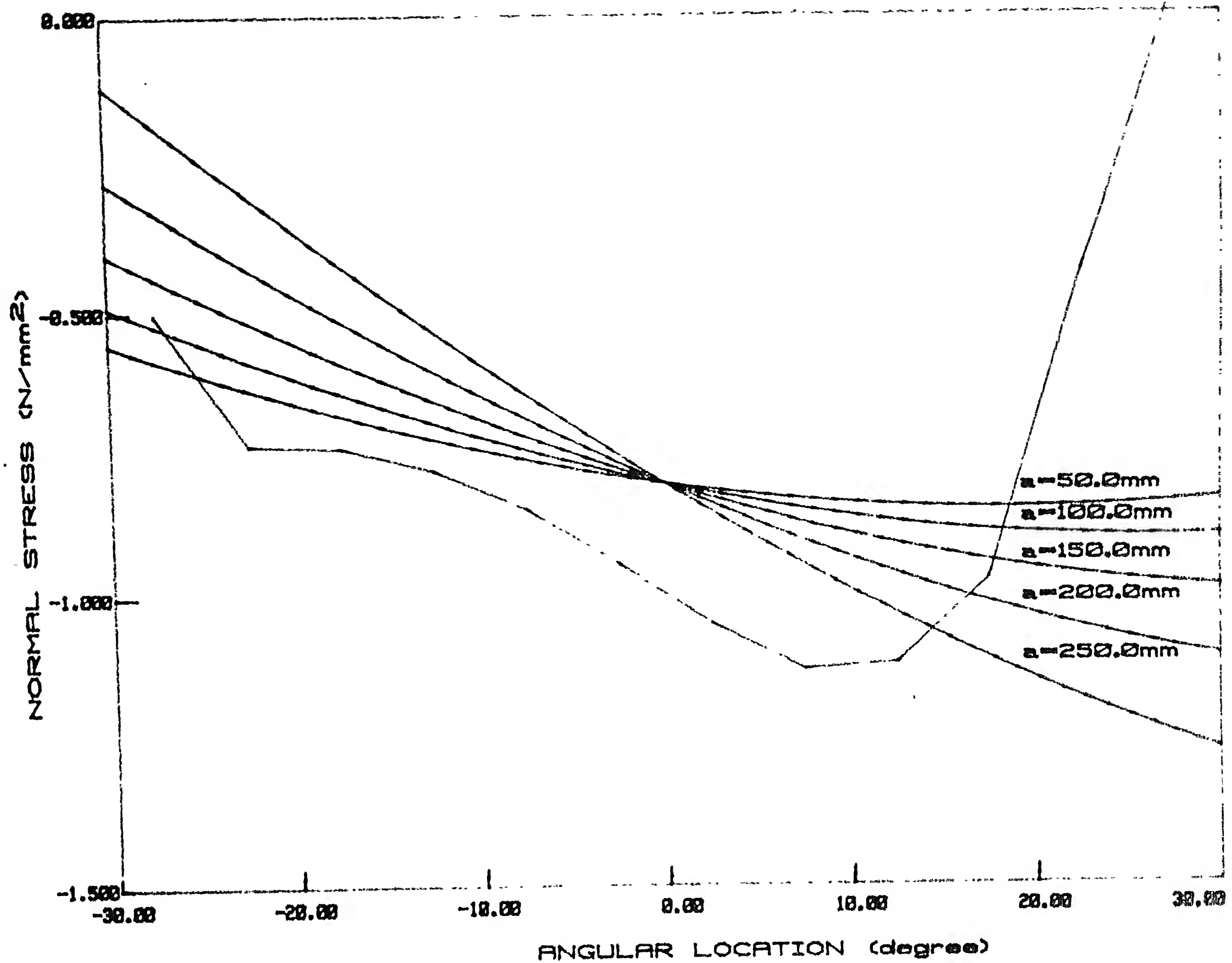
The drum is discritised using 168 elements and the shoe has 72 elements . 13 nodes are initially assumed to be in contact .

3.3 Contact Stresses And Slip

Fig. 3.3 shows the normal stress variation over the contact surface . As expected , the variation has a peak to the right of the mid point of the shoe . To compare the variation and the value of the maximum normal stress with the corresponding relations used in the commercial brake design , eqn 1.3 is plotted in fig. 3.4



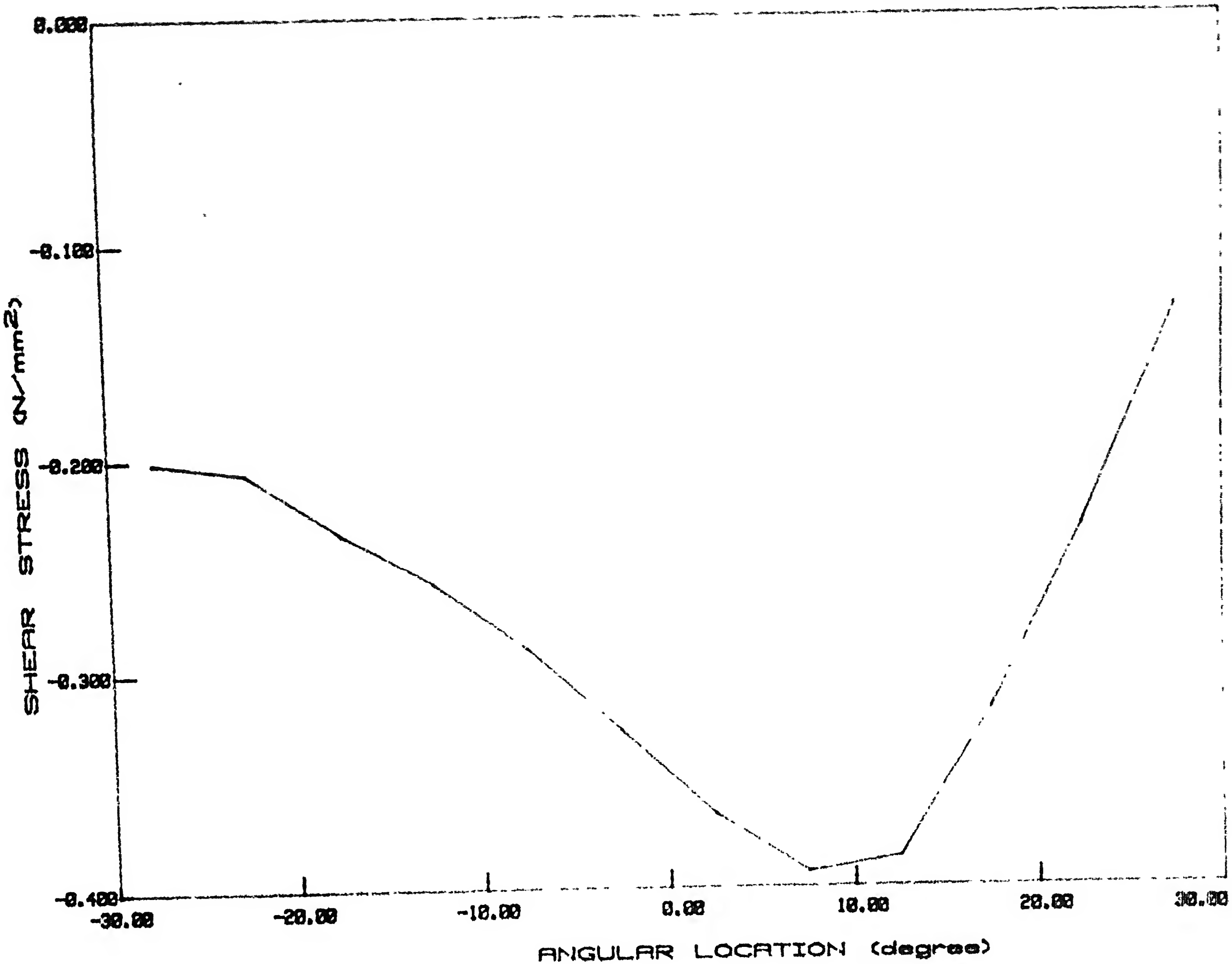
3.3 NORMAL STRESS VARIATION FOR REF. CONFIGURATION



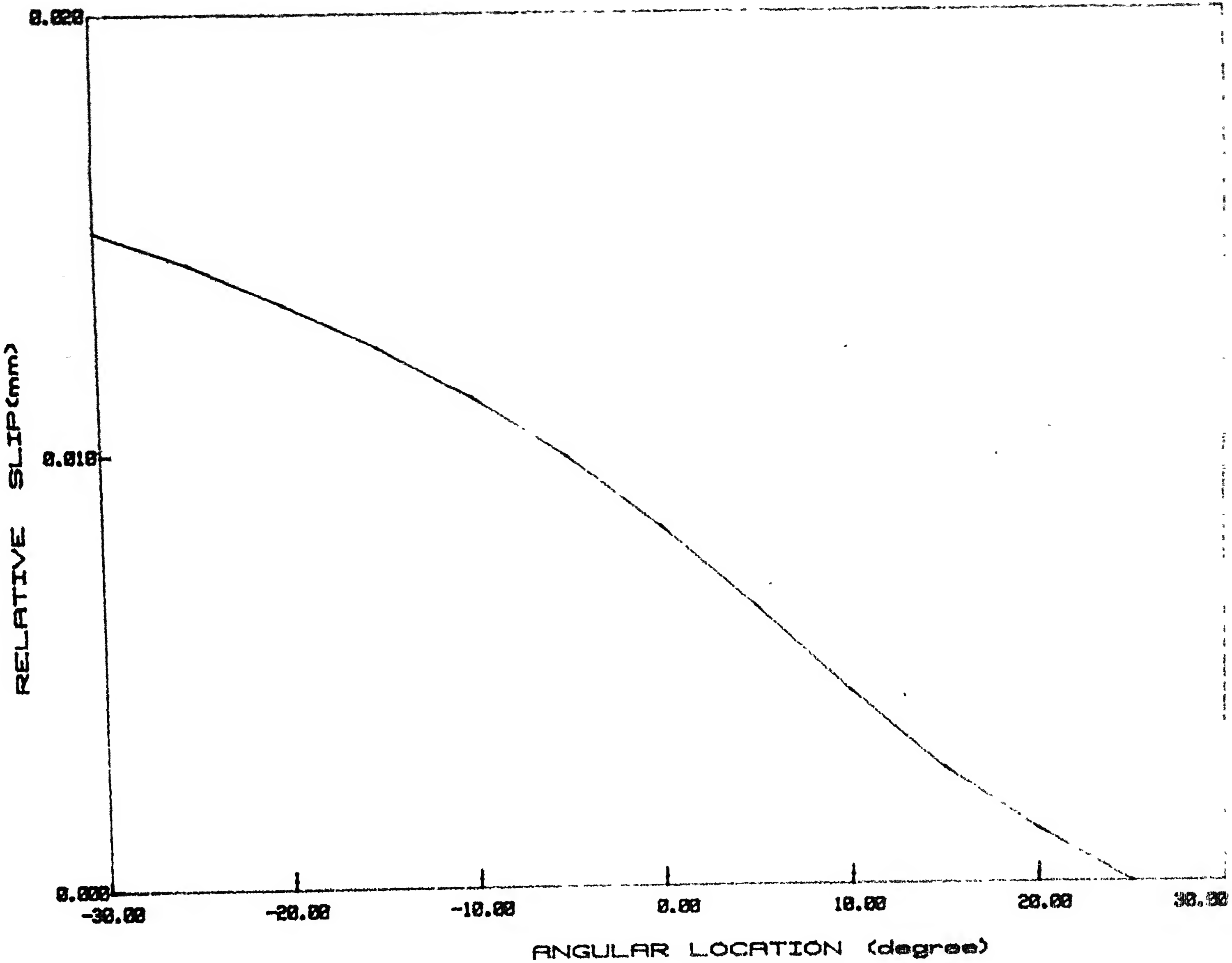
3.4 COMPARISON WITH THE RESULTS OF CONVENTIONAL DESIGN

along with our results . As the equation depends on the lever dimension 'a' (see fig. 1.1) , it is plotted for several values of 'a' in a certain range . As stated in chapter 1 , this equation has been derived by assuming the pressure to be proportional to the wear and by neglecting the elasticity of the drum in estimating the wear variation . On the other hand our results are for a new brake for which the wear is zero and which take into account the elasticity of the drum . Further , in our formulation the pressure is independent of the lever dimension 'a' . So it is expected that these two graphs should not match . However , two points need to be made . First is that according to eqn (1.4) , 'p' doesn't drop to zero at the last contact point . This is not realistic . Secondly for typical values of 'a' , eqn. (1.4) underestimates the value of p_{max} for a new brake . Since this value of p_{max} is used in choosing the other brake parameters in the conventional method , the method seems to give very conservative design .

The variation of shear stress over the contact surface is shown in fig. 3.5 . Since all the contact nodes slip , the shear stress should be μ times the normal stress . Thus the variation of the shear stress should be the same as that of the normal stress . By comparing the fig. 3.3 and 3.5 , it is found that the variation is more or less same . But at some points , the values of shear stress exceed the product of μ times the normal stress by a



3.5 SHEAR STRESS VARIATION FOR REF. CONFIGURATION



3.6 SLIP VARIATION FOR REF. CONFIGURATION

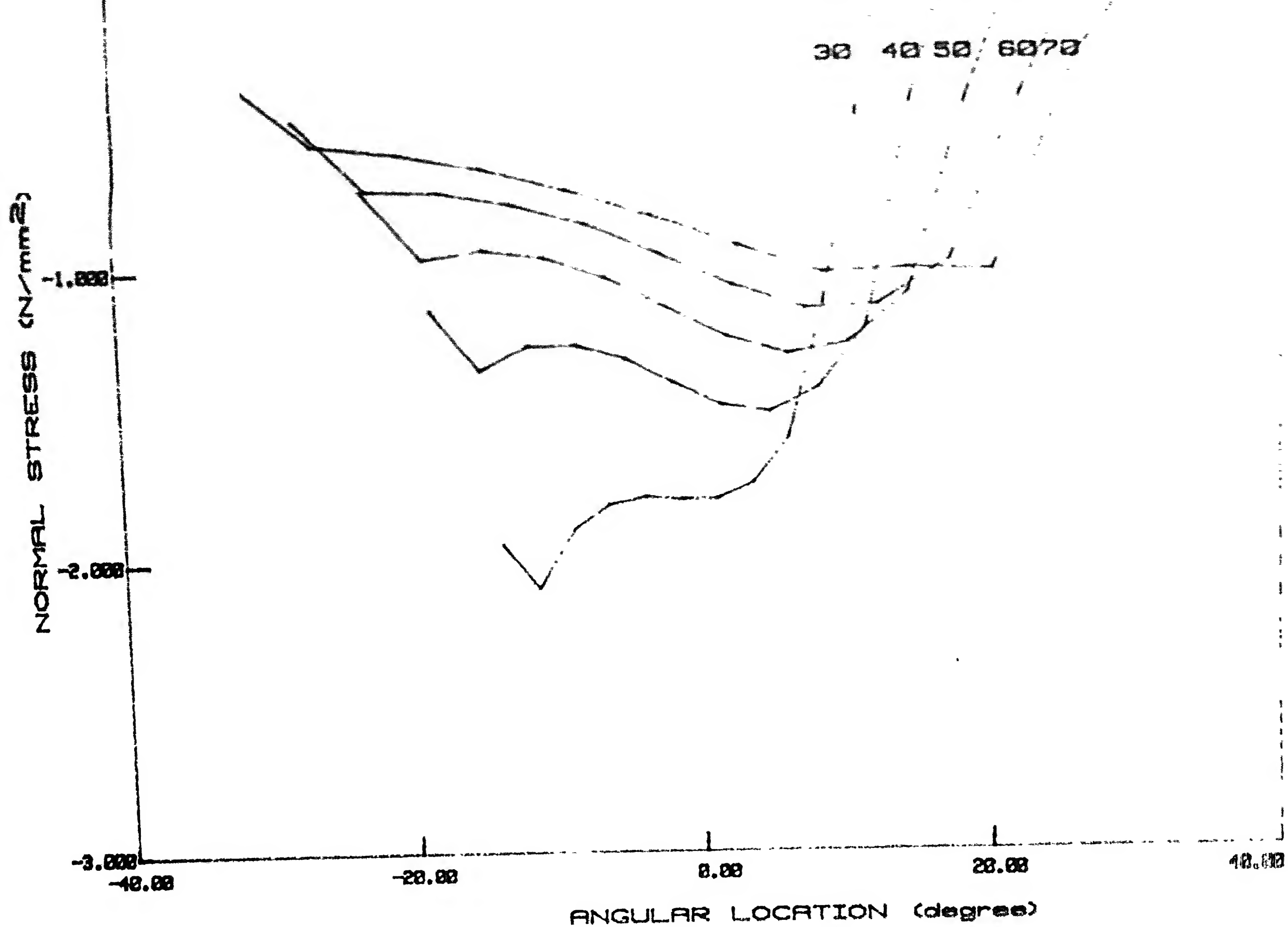
considerable amount . This discrepancy may be attributed to numerical error as well as to the fact that the coulomb relation was applied to the nodal forces rather than the stresses .

The variation of relative slip is plotted in fig. 3.6 . It is observed that the relative slip is maximum at the left - most contact point and decreases gradually to zero at the right most contact point . Fig. 3.3 and 3.6 together show that , except at the end - points , product of the pressure and the slip is more or less uniform . Thus , at least for a new brake , the rate of wear will be uniform over most of the contact region .

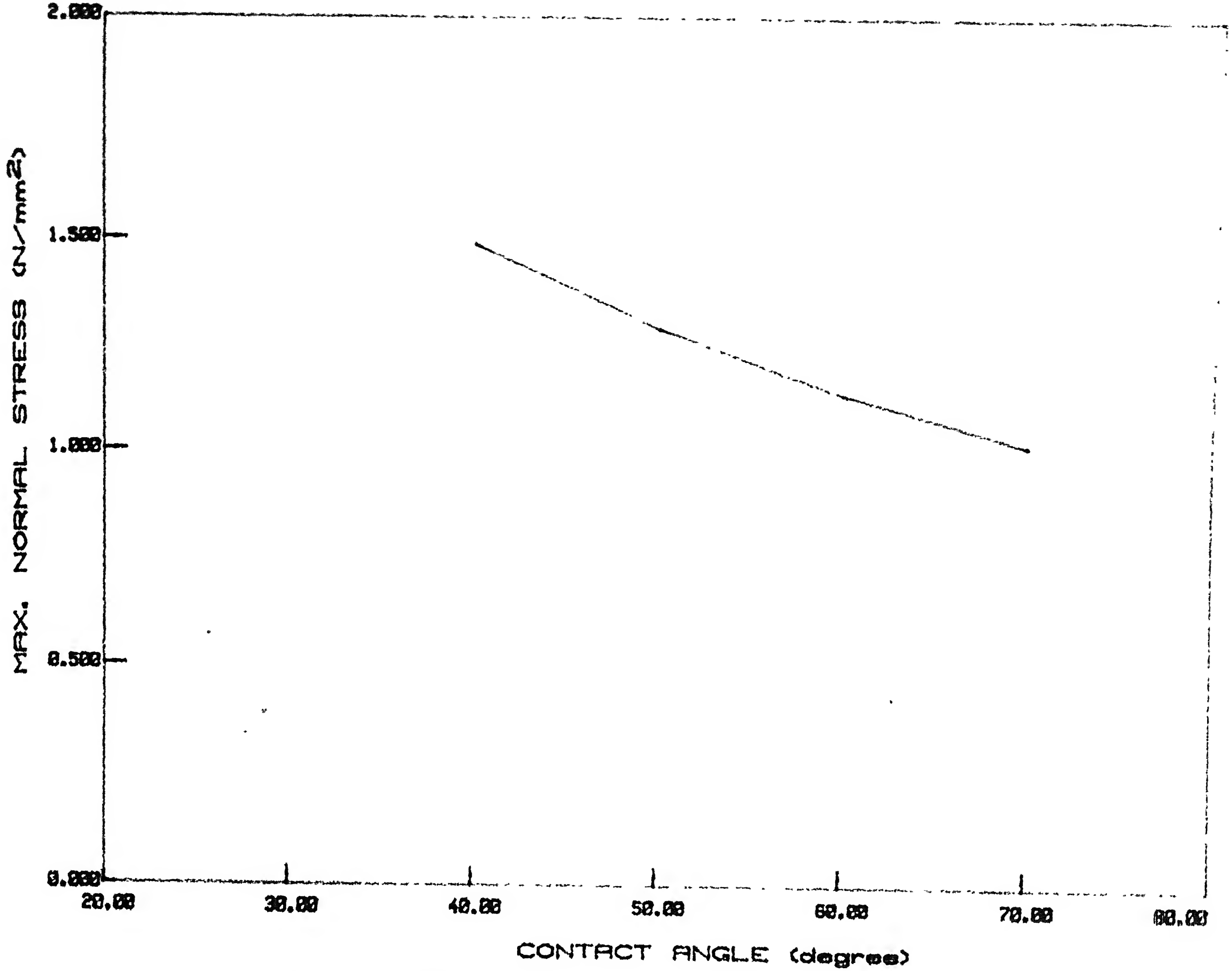
3.4 Parametric Studies

(a) Contact Angle

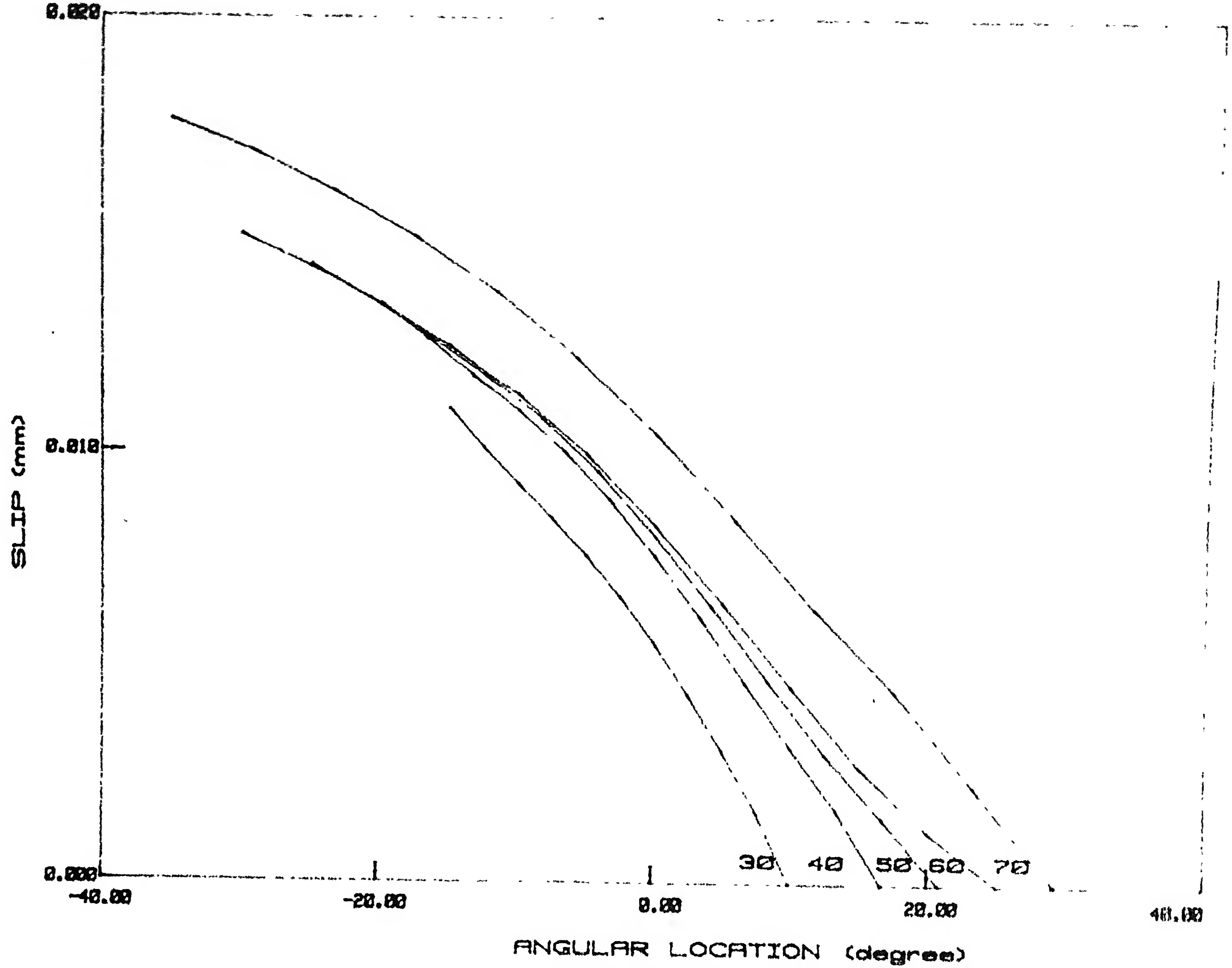
Fig. 3.7 shows the variations of the normal stress along the central surface for various contact angles . The pattern of the variation doesn't change with the contact angle but the value of maximum normal stress decreases with the contact angle . This variation is shown in fig. 3.8 . This is expected , as smaller contact angle means smaller contact area . But since the total normal force remains the same , the maximum value has to increase . Fig. 3.9 shows that although the variation of slip doesn' change with the contact angle , it's value increases



3.7 VARIATION OF NORMAL STRESS FOR DIFFERENT CONTACT ANGLES



3.8 VARIATION OF MAX. NORMAL STRESS WITH TOTAL CONTACT ANGLE



3.9 VARIATION OF SLIP FOR DIFFERENT CONTACT ANGLES

In conventional design procedure , once the material is selected , the contact area is chosen from the condition that p_{max} should not exceed the capacity of the material to withstand certain pressure . From the above results , we can conclude that it is disadvantageous to choose a contact length longer than the required , as it would increase the frictional work considerably .

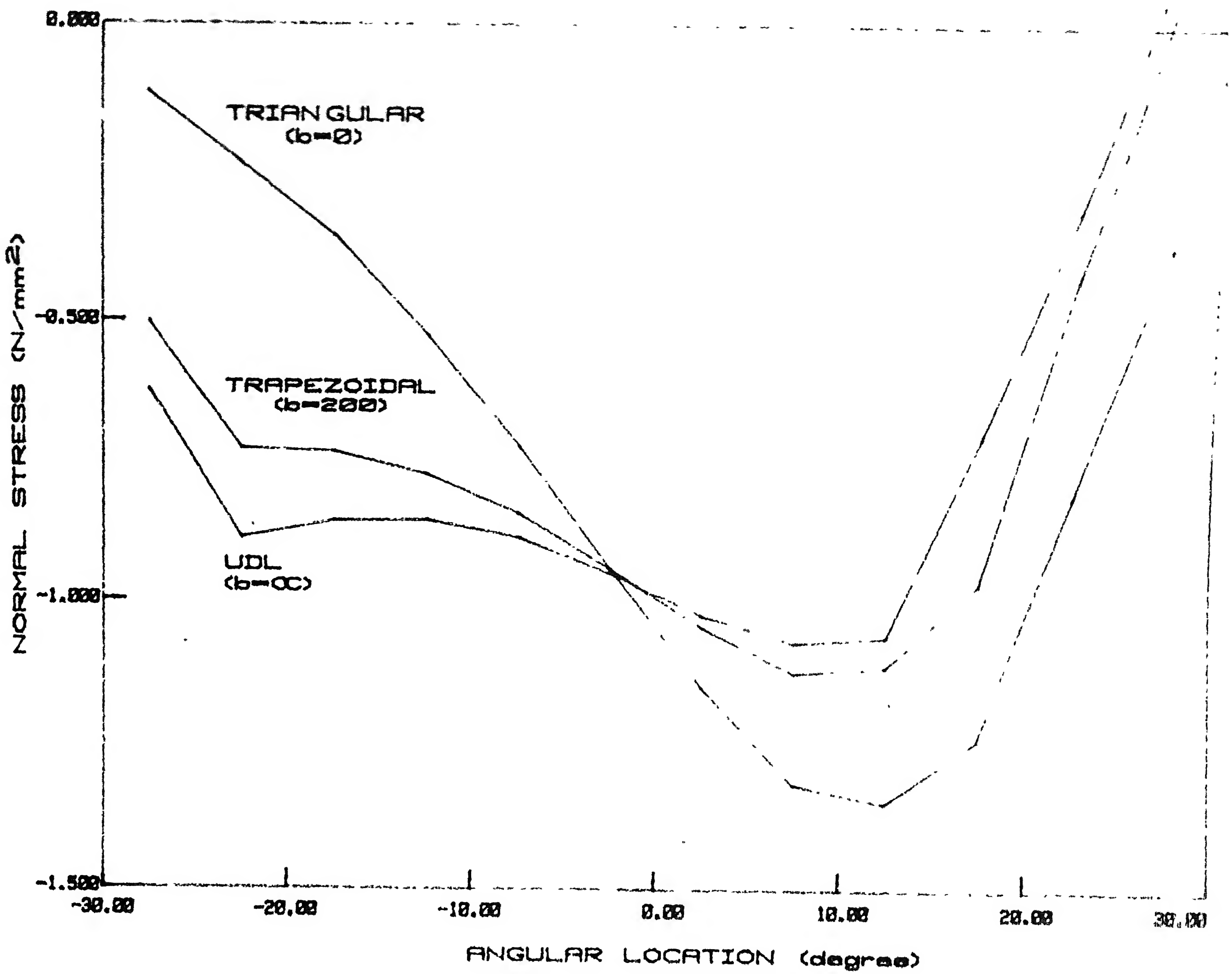
(b) Coefficient of friction

Varying the coefficient of friction has very little effect on either the variation or the values of the normal stress . The shear stress values , however , increase proportionally with μ . Further , the values or the variation of slip also do not change appreciably .

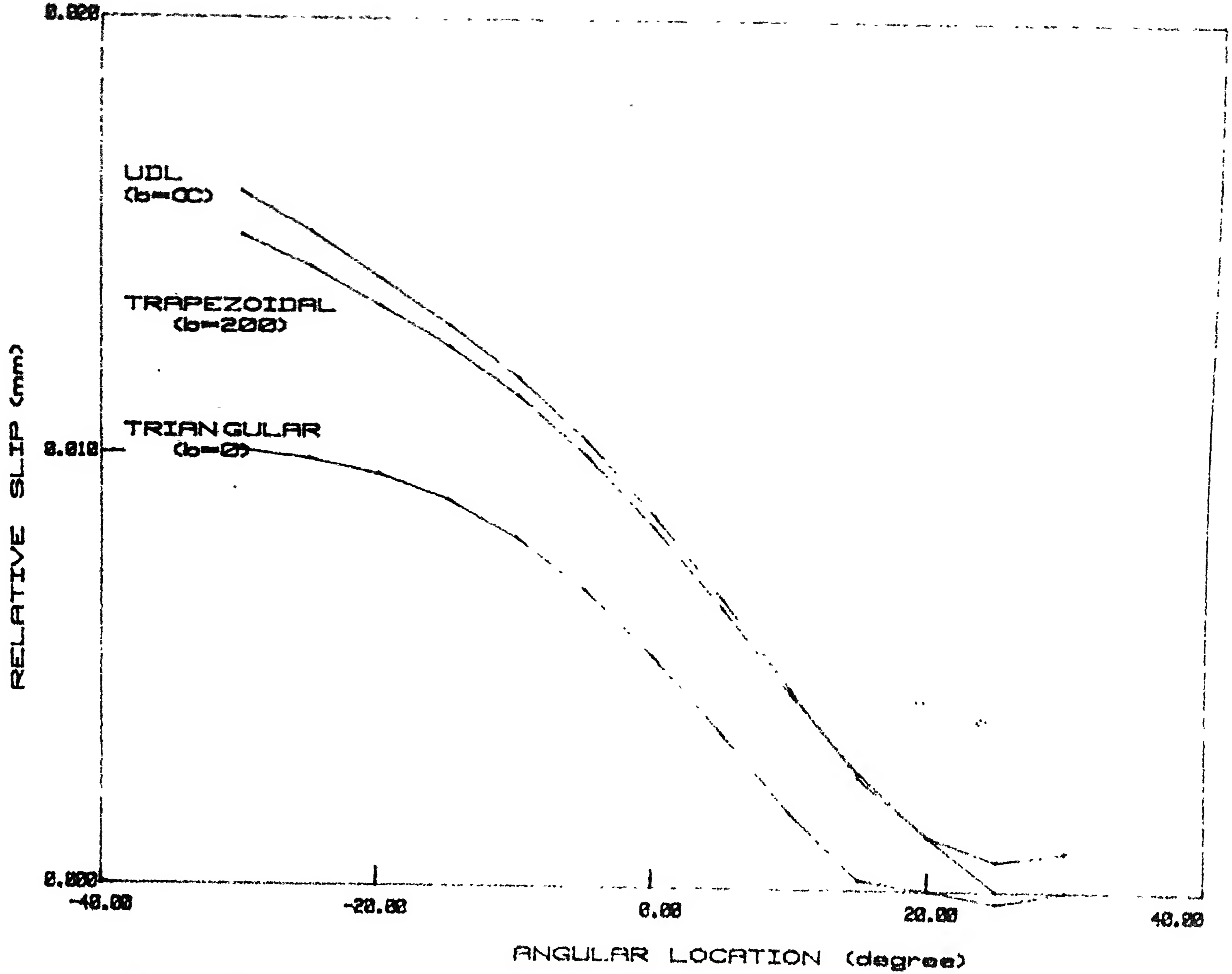
If it is possible to increase the coefficient of friction of the shoe material without changing it's capacity for sustaining a certain pressure , then it is desirable to do so . In that case one needs smaller contact area to generate the same braking torque .

(c) Hinge Location

Fig. 3.10 shows the variation of the normal stress along the contact length for three different hinge locations . The parameter used to characterize the hinge location is 'b' , the



3.10 VARIATION OF NORMAL STRESS FOR DIFFERENT FORCE DISTRIBUTION



3.11 VARIATION OF SLIP FOR DIFFERENT FORCE DISTRIBUTION

horizontal distance of the left edge of shoe from the fulcrum . The first graph is for one extreme case of b=0 (i.e. triangular variation of normal stress) while the third graph is for the other extreme case of b->infinity (i.e. uniform variation of the normal stress) . The variation of slip for these three values of 'b' is shown in fig. 3.11 .

While the variation of slip is not affected by change in 'b' , that of the normal stress does . For a triangular loading (b=0) , the variation of the normal stress is also triangular . It is observed that for b>>200 , there is hardly any decrease in the value of maximum normal stress . So no advantage is gained by increasing b beyond this . Instead , the space requirement of the braking system would be more .

CHAPTER 4

Conclusion And Suggestion For Future Work

4.1 Conclusion

The following conclusions can be drawn regarding the analysis of brake - drum assembly as a two body contact problem :-

(1) The algorithm which so far has been restricted to only straight contact surface and normal loading can be applied to curved contact surface and tangential loading as well .

(2) The slip at the contact surface is not uniform . However the product of the normal stress and slip is more or less uniform over most of the contact surface . Therefore , the wear will be uniform .

(3) Increasing the whole contact angle increases the amount of slip .

(4) The normal stresses and slip are unaffected by change in the coefficient of friction .

(5) Changing the distance of the fulcrum from the drum center beyond a certain value has no effect either on the normal stresses or slip .

4.2 Suggestion for Future Work

The following suggestions are made for future work :-

(a) The analysis can be improved by including the body - forces due to rotation of the drum .

(b) The application of variable coefficient of friction , which depends on factors like the rise in temperature of shoe , the drum velocity , the wear of shoe etc. ,will give a better estimate of maximum contact stresses or slip .

(c) Since the exact stress boundary condition on the top part of the shoe are difficult to estimate , it is better to include the lever in the analysis to make it a three - body contact problem . To analyze such a problem , the present contact algorithm will have to be modified .

(d) In our formulation , the normal stresses can depend on the wear only through the change in contact geometry of the shoe . To take care of this dependence , one can develop an incremental algorithm in which the wear is calculated at periodic intervals of time and used to update the contact geometry of the shoe .

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1. [illegible]
2. Finite element methods